

NAG Toolbox for MATLAB

s19ac

1 Purpose

s19ac returns a value for the Kelvin function $\ker x$, via the function name.

2 Syntax

```
[result, ifail] = s19ac(x)
```

3 Description

s19ac evaluates an approximation to the Kelvin function $\ker x$.

Note: for $x < 0$ the function is undefined and at $x = 0$ it is infinite so we need only consider $x > 0$.

The function is based on several Chebyshev expansions:

For $0 < x \leq 1$,

$$\ker x = -f(t) \log x + \frac{\pi}{16} x^2 g(t) + y(t)$$

where $f(t)$, $g(t)$ and $y(t)$ are expansions in the variable $t = 2x^4 - 1$.

For $1 < x \leq 3$,

$$\ker x = \exp\left(-\frac{11}{16}x\right) q(t)$$

where $q(t)$ is an expansion in the variable $t = x - 2$.

For $x > 3$,

$$\ker x = \sqrt{\frac{\pi}{2x}} e^{-x/\sqrt{2}} \left[\left(1 + \frac{1}{x} c(t)\right) \cos \beta - \frac{1}{x} d(t) \sin \beta \right]$$

where $\beta = \frac{x}{\sqrt{2}} + \frac{\pi}{8}$, and $c(t)$ and $d(t)$ are expansions in the variable $t = \frac{6}{x} - 1$.

When x is sufficiently close to zero, the result is computed as

$$\ker x = -\gamma - \log\left(\frac{x}{2}\right) + \left(\pi - \frac{3}{8}x^2\right) \frac{x^2}{16}$$

and when x is even closer to zero, simply as $\ker x = -\gamma - \log\left(\frac{x}{2}\right)$.

For large x , $\ker x$ is asymptotically given by $\sqrt{\frac{\pi}{2x}} e^{-x/\sqrt{2}}$ and this becomes so small that it cannot be computed without underflow and the function fails.

4 References

Abramowitz M and Stegun I A 1972 *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

5.1 Compulsory Input Parameters

1: **x** – double scalar

The argument x of the function.

Constraint: $x > 0$.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **result** – double scalar

The result of the function.

2: **ifail** – int32 scalar

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail = 1

On entry, x is too large: the result underflows. On soft failure, the function returns zero.

ifail = 2

On entry, $x \leq 0$: the function is undefined. On soft failure the function returns zero.

7 Accuracy

Let E be the absolute error in the result, ϵ be the relative error in the result and δ be the relative error in the argument. If δ is somewhat larger than the *machine precision*, then we have:

$$E \simeq \left| \frac{x}{\sqrt{2}} (\ker_1 x + \operatorname{kei}_1 x) \right| \delta,$$

$$\epsilon \simeq \left| \frac{x}{\sqrt{2}} \frac{\ker_1 x + \operatorname{kei}_1 x}{\ker x} \right| \delta.$$

For very small x , the relative error amplification factor is approximately given by $\frac{1}{|\log x|}$, which implies a strong attenuation of relative error. However, ϵ in general cannot be less than the *machine precision*.

For small x , errors are damped by the function and hence are limited by the *machine precision*.

For medium and large x , the error behaviour, like the function itself, is oscillatory, and hence only the absolute accuracy for the function can be maintained. For this range of x , the amplitude of the absolute

error decays like $\sqrt{\frac{\pi x}{2}} e^{-x/\sqrt{2}}$ which implies a strong attenuation of error. Eventually, $\ker x$, which asymptotically behaves like $\sqrt{\frac{\pi}{2x}} e^{-x/\sqrt{2}}$, becomes so small that it cannot be calculated without causing

underflow, and the function returns zero. Note that for large x the errors are dominated by those of the standard function EXP.

8 Further Comments

Underflow may occur for a few values of x close to the zeros of $\ker x$, below the limit which causes a failure with **ifail** = 1.

9 Example

```
x = 1;
[result, ifail] = s19ac(x)

result =
    0.2867
ifail =
     0
```
